THE FIFTH INTERNATIONAL MATHEMATICAL OLYMPIAD
Blagoveshchensk – Russia, 15 March 2025

Problem 1 ***(10 points)***
Find the distance between the graphs of the functions

$$f_1(x) = \frac{\ln x}{2025} \quad \text{and} \quad f_2(x) = e^{2025x}.$$

Problem 2 ***(10 points)***
Calculate the area of the figure bounded by the curve loop $x^3 + x^2 - y^2 = 0$.

Problem 3 ***(10 points)***
Calculate the double integral

$$\iint_D (x + y)dxdy,$$

where the region D is bounded by curves $xy = 1$, $xy = 9$, $y - x = 2$, $x - y = 2$ $(x > 0, y > 0)$.

Problem 4 ***(8 points)***
Find $x, y \in R$ that satisfy the system of equations

$$\begin{cases} 34x^2 - 22xy + 5y^2 = 98, \\ 16x^2 + 2xy - 3y^2 = 0. \end{cases}$$

Problem 5 ***(10 points)***
Calculate the limit of the sequence

$$\lim_{n\to\infty} n^2 \cdot \left(\sqrt[n]{n + a^2} - \sqrt[n]{n}\right).$$

Problem 6 ***(8 points)***

Find x at which $\Delta(x) = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 4 & 2 & 1 & 2 & 0 \\ 8 & 4 & 2 & 2 & 1 \\ x & -8 & -4 & -4 & -1 \end{vmatrix} = 0.$

Problem 7 ***(12 points)***

Find a general solution to the differential equation

$$x^2 y y'' - 2x^2 {y'}^2 + x y y' + y^2 = 0.$$

Problem 8 ***(12 points)***

Find a general solution to the system of differential equations

$$\begin{cases} 2zy' = y^2 - z^2 + 1, \\ z' = z + y. \end{cases}$$

Problem 9 ***(9 points)***

Calculate $y^{(21)}(0)$ for the function

$$y = e^{2x} \sin 2x.$$

Problem 10 ***(11 points)***

How many times do you need to throw three dice so that the probability that at least one of the throws will produce a combination with three identical numbers (for example, 1-1-1, 2-2-2, etc.) becomes greater than a given number p, where $0 < p < 1$? How many throws will be required if $p = 0.5$?